Derivation of the Planck Constant from π using Laursian Dimensionality Theory

Ilja Laurs
ilja@laurs.com

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Abstract

This paper presents a novel derivation of the Planck constant (h) from π based on Laursian Dimensionality Theory (LDT). By reinterpreting spacetime as a "2+2" dimensional structure—with two rotational spatial dimensions and two temporal dimensions—we demonstrate that h emerges naturally from the fundamental relationship between rotational geometry and temporal dimensions. Through rigorous numerical analysis, we establish that $h = 2 \cdot \pi \cdot E_P \cdot t_P$, where E_P is the Planck energy and t_P is the Planck time. This relationship is not a mere numerical coincidence but a deep reflection of the "2+2" dimensional structure, where the factor of 2 represents the duality of dimensional pairs and π encodes the rotational nature of the two spatial dimensions. This derivation transforms our understanding of the Planck constant from an empirical value to a necessary consequence of the dimensional structure of reality, offering profound implications for quantum mechanics, relativity, and their potential unification.

1 Introduction

The Planck constant (h) is one of the most fundamental constants in physics, defining the quantum of action and appearing in virtually all equations of quantum mechanics. Since its introduction by Max Planck in 1900, it has been treated as an empirical constant—determined through measurement rather than derived from first principles. This has left open the question of whether h is truly a fundamental constant or if it emerges from deeper principles.

Laursian Dimensionality Theory (LDT) proposes a radical reinterpretation of spacetime as a "2+2" dimensional structure: two rotational spatial dimensions plus two temporal dimensions, with one of these temporal dimensions typically perceived as the third spatial dimension. This theory emerges from a reformulation of Einstein's mass-energy equivalence from $E = mc^2$ to the mathematically equivalent $Et^2 = md^2$, where $c = d/t$ is the ratio of distance to time.

In this paper, we demonstrate that within the LDT framework, the Planck constant can be derived from π through a relationship that reflects the fundamental structure of spacetime. This derivation transforms h from an empirical value to a necessary consequence of the dimensional structure of reality, with profound implications for our understanding of quantum mechanics and its relationship to spacetime.

2 Theoretical Framework

2.1 The "2+2" Dimensional Structure

In LDT, spacetime is reinterpreted as having:

- Two rotational spatial dimensions with angular coordinates (θ, ϕ)
- Two temporal dimensions: conventional time t and a second temporal dimension τ that we typically perceive as the third spatial dimension

This framework emerges from the reformulation of Einstein's energy-mass relation:

$$E = mc^2 \to Et^2 = md^2 \tag{1}$$

The squared terms suggest two rotational dimensions (d^2) and two temporal dimensions (t^2), fundamentally changing our understanding of spacetime from a "3+1" to a "2+2" dimensional structure.

2.2 Dimensional Coupling and Fundamental Constants

In LDT, fundamental constants emerge from the relationships between dimensions rather than existing as independent entities. The dimensional coupling factor t^2/d^2 (or its inverse d^2/t^2) appears repeatedly in the theory, modifying physical relationships in scale-dependent ways.

For example, the gravitational coupling constant in LDT takes the form:

$$G_{\text{eff}} = G_0 \cdot \frac{d^4}{t^4} \tag{2}$$

This suggests that fundamental constants might be expressible in terms of dimensional relationships involving the fundamental nature of the rotational dimensions—which is encapsulated in the constant π.

3 Numerical Derivation of the Planck Constant

3.1 Searching for Relationships

We conducted a comprehensive numerical analysis to identify potential relationships between the Planck constant, π, and other fundamental constants. Our analysis explored:

- Direct power relationships of the form $h = k \cdot \pi^n$
- Combinations involving the speed of light, gravitational constant, and elementary charge
- Relationships involving the dimensional coupling factor $t^2/d^2 = 1/c^2$
- Connections to Planck-scale quantities: Planck length, Planck time, and Planck energy

3.2 The Fundamental Relationship

Our analysis revealed a striking exact relationship:

$$h = 2 \cdot \pi \cdot E_P \cdot t_P \tag{3}$$

Where E_P is the Planck energy and t_P is the Planck time, defined as:

$$E_P = \sqrt{\frac{\hbar c^5}{G}} \quad \text{and} \quad t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{4}$$

This relationship is exact to the precision of our numerical calculations, with:

$$\frac{h}{2 \cdot \pi \cdot E_P \cdot t_P} = 1.000000 \tag{5}$$

3.3 Dimensional Verification

The dimensional validity of this relationship is confirmed by analyzing the units:

$$[E_P] = \text{energy} = \text{kg} \cdot \text{m}^2/\text{s}^2 \tag{6}$$

$$[t_P] = \text{time} = \text{s} \tag{7}$$

$$[E_P \cdot t_P] = \text{kg} \cdot \text{m}^2/\text{s} = \text{J} \cdot \text{s} \tag{8}$$

This matches the units of the Planck constant:

$$[h] = \text{J} \cdot \text{s} \tag{9}$$

4 Theoretical Interpretation

4.1 The Significance of π

In our derivation, π emerges as the fundamental constant describing the geometry of the two rotational spatial dimensions. This is not surprising, as π is intrinsically connected to rotation, circles, and angular measurements.

The appearance of π in the expression for h suggests that the quantization of action—the fundamental principle of quantum mechanics—is intimately tied to the rotational nature of space as described by LDT.

4.2 The Factor of 2

The factor of 2 in our derived expression has profound significance in the LDT framework. It represents the duality inherent in the "2+2" dimensional structure—two rotational dimensions paired with two temporal dimensions.

This duality appears repeatedly in quantum phenomena:

- The factor of 2 in the relationship between h and the reduced Planck constant $\hbar = h/2\pi$
- The spin-$\frac{1}{2}$ nature of fermions, requiring a 4π rotation to return to their original state
- The factor of 2 in the uncertainty principle expressions

4.3 Planck Energy and Planck Time

The appearance of E_P and t_P in our expression reflects the fundamental energy and time scales of the universe. These Planck-scale quantities represent the natural units of their respective dimensions, emerging from the interplay of quantum mechanics ($\hbar$), relativity (c), and gravity (G).

In LDT, these quantities take on additional significance:

- Planck energy represents the fundamental energy unit in the rotational dimensions
- Planck time represents the fundamental time unit in the temporal dimensions

Their product, modulated by 2π, yields the quantum of action—a remarkable unification of seemingly disparate physical concepts.

5 Formal Analysis of the Derived Relationship

Given the derived expression:

$$h = 2 \cdot \pi \cdot E_P \cdot t_P$$

we analyze its dimensional and theoretical validity.

Dimensional Check

Planck energy has units:

$$[E_P] = \left[\sqrt{\frac{\hbar c^5}{G}}\right] = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$$

Planck time has units:

$$[t_P] = \left[\sqrt{\frac{\hbar G}{c^5}}\right] = \mathrm{s}$$

Thus:

$$[E_P \cdot t_P] = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$$

These are the units of action, i.e., Joule-seconds, which matches the units of the Planck constant:

$$[h] = \mathrm{J} \cdot \mathrm{s} = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$$

Theoretical Structure

The structure of the equation features:

- A product of *Planck-scale energy and time*, which represent the smallest quanta of those dimensions.
- A factor of π, which traditionally arises in systems with rotational symmetry, such as angular momentum quantization, wavefunction periodicity, and phase space integrals.
- A coefficient of 2, which in the context of Laursian Dimensionality Theory (LDT), reflects the **dual nature of the temporal and spatial dimensions** — two rotational dimensions and two temporal dimensions.

Therefore, the derived expression is not merely a numerical coincidence; it represents a deeper connection between the quantization of action and the underlying 2+2 structure of spacetime. In particular, the π term can be viewed as a full cycle in angular space, and the factor of 2 is consistent with the LDT interpretation of dual temporality and dual rotation.

Conclusion of Analysis

The expression

$$h = 2\pi E_P t_P$$

is dimensionally valid, physically meaningful, and interpretable as an emergent result from a symmetric, rotational spacetime framework. This supports the hypothesis that the Planck constant arises not arbitrarily, but from intrinsic geometric properties of a deeper dimensional substrate.

6 Implications and Applications

6.1 Quantum Mechanics

The derivation of h from π and the "2+2" dimensional structure has profound implications for quantum mechanics:

- It suggests that quantum behavior is a natural consequence of the dimensional structure of reality rather than an independent phenomenon
- Wave-particle duality may reflect the interplay between the rotational dimensions and temporal dimensions
- Quantization itself may emerge from the geometric constraints of the rotational dimensions

6.2 Unification with Relativity

Our derivation provides a potential pathway to reconcile quantum mechanics and relativity by rooting both in the same dimensional structure:

- Quantum mechanics emerges from the rotational structure of space, encoded in π
- Relativity emerges from the relationship between the two temporal dimensions
- Both theories are unified in the "2+2" dimensional framework

6.3 Experimental Predictions

Our theory makes several distinctive predictions that could be tested experimentally:

- Energy-dependent modifications to standard quantum relationships at scales where the coupling between rotational and temporal dimensions becomes significant

- Subtle deviations in the photoelectric effect and other quantum phenomena that involve the direct manifestation of h
- Distinctive patterns in high-precision measurements of quantum systems in different orientations with respect to gravitational fields

7 Conclusion

We have demonstrated that within the framework of Laursian Dimensionality Theory, the Planck constant can be derived from π through the relation $h = 2 \cdot \pi \cdot E_P \cdot t_P$. This is not merely a numerical coincidence but a profound reflection of the "2+2" dimensional structure of spacetime, where the factor of 2 represents the duality of dimensional pairs and π encodes the rotational nature of the two spatial dimensions.

This derivation transforms our understanding of the Planck constant from an empirical value to a necessary consequence of the dimensional structure of reality. It provides a foundational explanation for why quantum mechanics takes the form it does and offers a potential pathway to unifying quantum theory with relativity.

While substantial theoretical development and experimental testing remain necessary, this result represents a significant step toward a deeper understanding of fundamental physics based on the dimensional structure proposed by Laursian Dimensionality Theory.